How does changing the slit separation of a two-point source affect the distance between light fringes?

Introduction:

Light is known to have both the properties of a wave and of a particle. This is known as the wave particle duality. This concept is one of the most important concepts in studying quantum mechanics because it expresses that it is not enough to qualify quantum-scale objects solely as waves or particles (Harris and Freudenrich). Albert Einstein, one of the pioneers in the development of this idea, quoted "We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do." (Einstein and Infeld 262-263) This is what really interested me, when reading about the wave particle duality: how opposites are needed to be able explain a greater idea. Maybe we as humans can learn from light and learn to work with our opposites to make a whole.

Background Information:

One of the experiments that deepened our understanding of the wave-like properties of light was an experiment done by Thomas Young in 1801, wherein Young passed sunlight reflected off a steering mirror and passed it through a very small hole in a piece of paper. The beam of light was then passed through another card, this time with two slits. This resulted in a specific pattern of lines of light separated by areas of no light. Similarly, this investigation used a laser pointer, diffraction gratings, with differing number of lines to replicate the experiment and to observe the same results. Young's experiment was passed on the hypothesis that if light indeed had wave like properties then they would act like ripples in water: when the ripples meet, they should either act constructively and reinforce the ripples or act destructively and destroy each other.

This is known as wave interference. When the crests or troughs of two waves meet, meaning the waves are in phase, they should combine to make a bigger wave and when the trough and the crest of two waves meet or vice versa, meaning the waves are out of phase they should as destructively. This can be further explained by the principle of superposition, which can be stated as "When two waves interfere, the resulting displacement of the medium at any location is the algebraic sum of the displacements of the individual waves at that same location." This means that for example, at a given point, if a wave has a displacement of +2 units and another wave at the same point has a displacement of -1 units, the resulting wave will have a displacement of +1 unit due to the two waves interfering with each other.

What Young's experiment created was a two-point source of light that would interfere with each other to create a certain interference pattern. When the light created was put on a screen, areas or fringes of darkness and brightness were created. This was because, at the light fringes, the waves coming from the two sources were in-phase and acting constructively and at the dark fringes, the light waves were acting destructively. To find the equation that gives the exact position of these light fringes we must look at the geometry behind the diffraction of the waves and how the waves allow for constructive interference.

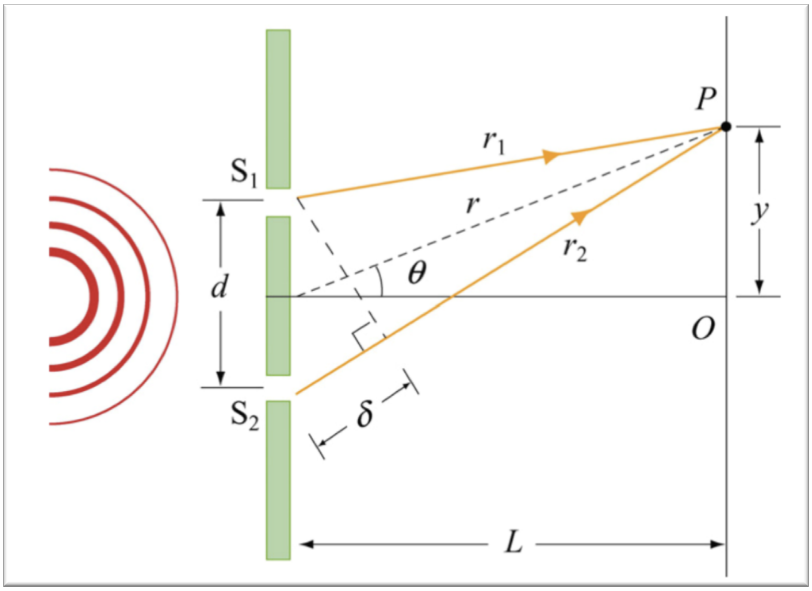


Figure 1: Double Slit Experiment (Liao 5)

Point P is the point where the light fringe falls, which is a distance y from the point O, perpendicular to the center of slit separation d and at a distance L from the screen. r_1 is the ray of light diffracted due to slit S_1, while r_2 is the ray of light diffracted due to slit S_2. δ is the extra distance that r_2 is required to move to reach point P compared to r_1. This is also known as the path difference of the two rays and $\delta = r_2 - r_1$. Finally, r is the shortest distance from the center of distance d. Using, the law of cosines to solve for both r_1 and r_2, two equations are found:

$$r_1{}^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr\cos\left(\frac{\pi}{2} - \theta\right) = r^2 + \left(\frac{d}{2}\right)^2 - dr\sin(\theta)$$

and

$$r_2{}^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr\cos\left(\frac{\pi}{2} + \theta\right) = r^2 + \left(\frac{d}{2}\right)^2 + dr\sin(\theta)$$

Subtracting the second equation from the first one creates

$$r_2{}^2 - r_1{}^2 = (r_2 + r_1)(r_2 - r_1) = 2dr\sin(\theta)$$

When, the distance L is much larger than the slit separation of d, r_1 and r_2 become approximately parallel (as seen in Figure 2) which yields a path difference of

$$\delta = (r_2 - r_1) \approx d \sin\theta$$

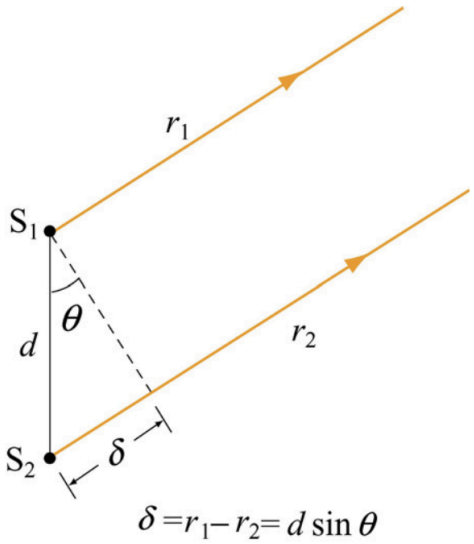


Figure 2: Path difference, when L is much greater than d (Liao 6)

Whether the two waves experience constructive or destructive, is determined by the value of the phase difference. When δ is equivalent to zero or if δ equivalent to an integer multiple of the wavelength of light, constructive interference occurs. This is because the two rays will in fact be parallel and the waves will be in-phase, meaning crests will meet crests and troughs will meet troughs. This results in the light fringes seen on the screen. This is seen in the equation below, where m is the order number. The order number is the number of light fringes after the central fringe which is given as $m = 0$

$$\delta = d \sin\theta = m\lambda, \qquad m = 0, \pm 1, \pm 2, \ldots (constructive)$$

Destructive interference then occurs when the phase difference is equivalent to an integer multiple of half the wavelength. When this occurs, the crest of one wave will meet the trough of the other or vice versa because it is half as long. This will result in destructive interference, creating dark fringes on the screen. The equation for this is:

$$\delta = d \sin\theta = (m + \frac{1}{2})\lambda, \qquad m = 0, \pm 1, \pm 2, \ldots (destructive)$$

Figure 3 shows how the difference in path difference can cause constructive and destructive interference.

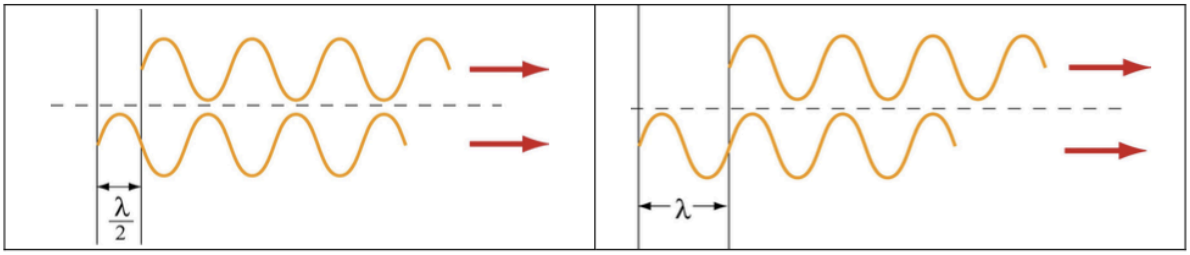


Figure 3: Destructive (left) and Constructive (right) Interference (Liao 7)

To find the distance between the first fringe and the central fringe, it must be assumed that the silt separation is much greater than the wavelength of light. This makes it so that the angle θ is very small. From that, using small angle approximation, the following equation is found, where y is equal to the distance from point O and point P as seen in Figure 1.

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

Substituting this equation into the constructive interference equation, a new equation is found to be:

$$\frac{dy}{L} = m\lambda$$

Finally, this research led to a hypothesis that increasing the slit separation, will decrease the distance between light fringes because when the formula is rearranged to isolate y, it becomes:

$$y = \frac{m\lambda L}{d}$$

This shows that increasing the slit separation will decrease the distance between the first fringe and the central fringe. (Liao 4-7)

Research Question: **How does changing the slit separation of a two-point source affect the distance between light fringes?**

Variables:

Independent Variable: Slit separation in m. Note: for this variable, the equation

$$\text{Slit Seperation} = \frac{1}{\text{Number of lines in the diffraction grating}}$$

was used. Also, the values given were chosen because these were the only diffraction gratings available.

Dependent Variable: Distance between light fringes in m. This was measured by taking the distance between the central fringe and the succeeding fringe.

Controlled Variables:

1. Distance between the diffraction gratings and the screen. This needs to be controlled because as seen in the equation $y = \frac{m\lambda L}{d}$, the distance between the diffraction gratings and the screen is directly proportional to the distance between the central and the first fringe. Increasing this distance would increase the distance between the central and the first fringes. This can be controlled by using a tape measure to clearly see that the screen is in the same place at all times.
2. Distance between diffraction grating and the laser source. This needs to be controlled because it is possible for the laser to diverge as the propagation distance increases. With that in mind, changing the distance between the laser source and the diffraction grating will cause this diversion to vary. This can be controlled by using the same tape measure and marking at which points the source and the screen are. This can also be controlled by using a collimating lens which will help make the waves parallel and reduce diversion.
3. Using the same light source. This needs to be controlled because like the distance between the diffraction gratings and the screen, this is directly proportional to the distance between the distance between the central and first fringe. Increasing the wavelength of the source would increase the distance between the central and the first fringes. This was controlled by using the same laser pointer for the whole investigation.

Materials and Methodology for Data Collection:

Materials:

- Diffraction Gratings (80, 300, 530, 527, 543, 600, lines per mm)
- Red Laser Source (650 nm wavelength)
- 2 Lab stands with Iron Clamps
- Railing
- Cardboard Screen
- 1 Meter Ruler ±0.0005m
- 30-cm Ruler ±0.0005m

Diagram for Set-up:

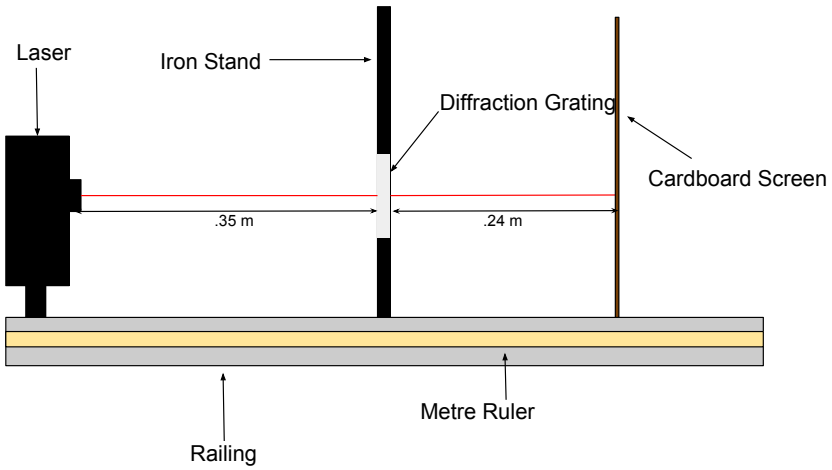


Figure 4: Diagram of Setup

Method:

1. Set up materials as seen in Figure 4.
2. Turn on the laser, and use the 80-line per mm diffraction grating.
3. Measure the distance between the central fringe and the next fringe, using the 30-cm ruler.
4. Repeat steps 2-3, 3 times for 3 trials per diffraction grating.
5. Repeat steps 2-4, for each of the diffraction gratings.

Safety Considerations:

1. Never look directly into the laser
2. Never point the laser directly into other people's eyes
3. Do not point the laser at reflective surfaces

Misuse of the laser can cause severe eye damage, which include flash blindness, afterimage, and glare. (UW EH&S)

Raw Data:

Table 1: Raw Quantitative Data Table for Slit Separation Against Distance Between Central and First Fringe			
	Length between central and first node (+/- 0.0005 m)		
Number of Lines per mm in the Diffraction Grating (lines mm^{-1})	Trial 1	Trial 2	Trial 3
80	0.0110	0.0120	0.0130
300	0.0430	0.0450	0.0450
520	0.0820	0.0800	0.0800
527	0.0850	0.0810	0.0810
543	0.0860	0.0830	0.0820
600	0.0930	0.0970	0.0960

Note: The uncertainty of the number of lines per mm in the diffraction grating is negligible because no measurements were taken to find this data, as the data was clearly labeled on the diffraction gratings themselves. Also for the diffraction gratings with 527 and 543 lines mm^{-1}, the actual number was given as lines $inch^{-1}$, so a conversion was done by dividing the given value by 25.4.

Processed Data:

Table 2: Processed Data Table for Slit Separation Against Distance Between Central and First Fringe			
Slit Separation (m)	1/Slit Separation (m^{-1})	Average length between central and first node (m)	Absolute uncertainty of average length (m)
1.25×10^{-5}	80000	0.0120	0.001
3.33×10^{-6}	300000	0.0443	0.001
1.92×10^{-6}	520000	0.0807	0.001
1.90×10^{-6}	527000	0.0823	0.002
1.84×10^{-6}	543000	0.0837	0.002
1.67×10^{-6}	600000	0.0953	0.002

Note: Because the uncertainty of the number of lines per mm in the diffraction grating is negligible, the slit separation and 1/slit separation will have negligible uncertainty as well.

Sample Calculations:

1. Average

$$Average = \frac{Sum\ of\ objects\ in\ a\ list}{Number\ of\ objects\ in\ a\ list}$$

$$Average\ length\ between\ central\ and\ first\ node = \frac{0.011 + 0.012 + 0.013}{3} = 0.012\ m$$

2. Slit Separation

$$Slit\ Separation = \frac{1}{Number\ of\ Lines\ per\ mm\ in\ the\ Diffraction\ Grating}$$

$$Slit\ Separation = \frac{1}{80} = 0.0125\ mm$$

This calculation needed to be done because the slit separation of each of the different diffraction gratings was not available. If 1 mm is divided by the number of lines per mm, the spacing in between each of the slits or the slit separation is what is left.

3. 1/Slit Separation

$$\frac{1}{Slit\ Separation} = 1 \div Slit\ Separation$$

$$\frac{1}{Slit\ Separation} = 1 \div 0.0125 = 80\ \text{mm}^{-1}$$

This piece of data was used to linearize the data found. The basic formula of $y = \frac{m\lambda L}{d}$ can be linearized by manipulating the formula into:

$$y = (\lambda L)D$$

Here, the *D* value equates to $\frac{1}{d}$ and is the *x* variable and λL is the slope.

Note: The *m* value is removed from the formula because the order value is equal to 1, as the node being measured is only the first node.

Error Propagation:

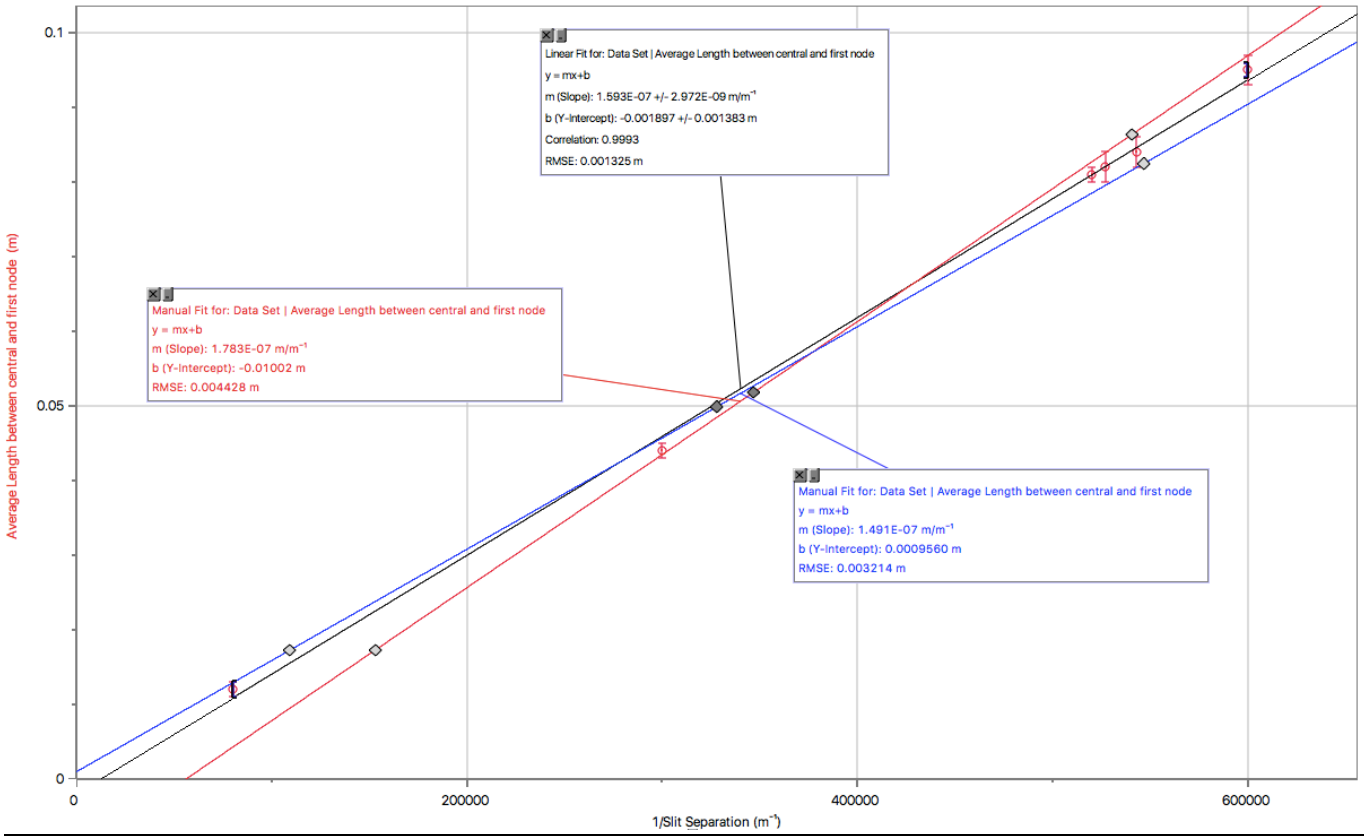
To take the absolute uncertainty of the average length between the central node and the first node, the formula below was used:

$$Absolute\ Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$Absolute\ uncertianty = \frac{0.013 - 0.011}{2} = 0.002\ m$$

Graph:

Graph 1: Graph For the Relationship between Average Length Between the Central and First Nodes and 1/Slit Separation



The given plot gives the relationship between the Average Length Between the Central and First Nodes and 1/slit separation. The graph shows that there is a linear relationship between the two variables, as the average length increases as the value for 1/Slit Separation increases. In turn, this means that as the slit separation increases, the average length decreases. This supports the initial hypothesis made. That being said, if the relationship between the two variables was truly directly proportional the slope of each of the points should be constant. Looking at points (80000,0.012), (300000,0.044), (520000, 0.081), their slopes are 1.50×10^{-7} m^2, 1.47×10^{-7} m^2, 1.56×10^{-7} m^2, respectively, thus showing that despite the very small variation between these numbers, they are not directly proportional. Furthermore, the line of best fit does not pass through the origin, further giving evidence that the relationship is not directly proportional

Lastly, the maximum and minimum lines were drawn and the values of these are 1.78×10^{-7} m^2 and 1.49×10^{-7} m^2, respectively. This gives the range of the slope as 1.59×10^{-7} $^{+1.9\times 10^{-8}}_{-1.0\times 10^{-8}}$ m^2.

Conclusion:

Based on the positive slope of the linear relationship between the average length between the central and first nodes and 1/slit separation, it can be said that as you increase the slit separation, the average length decreases, giving an inversely proportional relationship between the two variables. This makes sense because, making reference to Figure 1, for a given path difference (δ), increasing the d value will decrease the angle θ. This will in turn reduce the value $\frac{y}{L}$, based on the formula $\tan\theta = \frac{y}{L}$. Now, because the value of L is kept constant, the value of y or the length between nodes will decrease. This supports the hypothesis made. That being said, the relationship is not directly proportional as the slope at given points is not constant and the line does not pass through the origin.

Furthermore, the percent error of the found slope was found by using the formula:

$$\%\ Error = \frac{Literature\ Value - Experimental\ Value}{Literature\ Value} \times 100$$

In the case of this investigation, the literature value of the slope, which is equal to λL, is found by multiplying the given value of red laser with a wavelength of 650 nm / 6.5×10^{-7} m (UW EH&S) by the length between the screen and diffraction grating of 0.24 m. This gives a value of 1.56×10^{-7} m^2.

$$\%Error = \frac{1.56 \times 10^{-7} - 1.53 \times 10^{-7}}{1.56 \times 10^{-7}} \times 100 = 2\%$$

This shows that the data taken is very accurate.

Evaluation

Overall, there was very little error in the whole investigation. The only source of quantifiable uncertainty is in the use of the meter ruler in measuring the distance between the central and first nodes. This led to very small uncertainty in the y-axis as seen in Graph 1. This can be further reduced by using a more precise measuring device like a Vernier caliper.

Furthermore, since the distances between the central and first nodes were very small, it would have been more accurate to get the distance between the central node and the fourth node then divide the number taken by 4 to get the fringe separation. This was initially the proposed method but some of the diffraction gratings available only produced the central and the first node. With that in mind, this can be improved by using diffraction gratings of higher quality as the ones

used in this investigation were quite old and damaged. Damaged gratings could also cause the light passing through them to diffract differently due to the scratches on the diffraction gratings, causing more possible error.

A source of the systematic error found in the experiment could be due to the distortion and attenuation of light as it passed through the air. The investigation was not done in a vacuum and this causes the light to be attenuated, causing the wave of light from the laser to be affected. If resources are available, the experiment could be done in as much of a vacuum as possible. If not, reducing the distances between the source, lens, and screen will reduce the distance the light has to travel within the medium, which can reduce how much the light gets affected by attenuation. Furthermore, the use of a collimating lens will help reduce the level of diversion and attenuation that affects the ray of light.

As an extension, there are still 2 other factors that affect the fringe separation, which are the wavelength of light and the distance between the screen and the lens. These can be furthered explored by changing the color of the light, which changes its wavelength, and to move the screen closer or further with regard to the lens. Furthermore, this experiment can be done in different media and with different types of waves like with ripples in water or using sound waves.

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